Welcome to
Advanced Placement Calculus BC!!
Summer Math - 2023

As Advanced placement students, your first assignment for the 2023-2024 school year is to come to class the very first day in top mathematical form.

Calculus is a “world of change”. While words like limit, continuity, derivative and integral may seem foreign and daunting to you, I assure you that at this time next year at this time they will seem as commonplace to you as “graduation” and senioritis”!

In order to begin our journey, you must do some “mathercise” to keep your mind in shape through the summer months. So, I have prepared the attached assignment to help you “burn” your mathematical muscles! Make sure all of your work is presented in a logical order showing ALL of your work indicating your answers clearly. IF I CAN’T READ YOUR HANDWRITING, IT IS WRONG!!

This packet is a review of some Pre-calculus topics and some Calculus topics. It is to be done NEATLY and on a SEPARATE sheet of paper. Use your discretion as to whether you should use a calculator or not. When in doubt, think about if I would use one – that should guide you! Points will be awarded only if the correct work is shown, and that work leads to the correct answer. Assignment is due on the first full day of class September 6, 2023. Have a great summer and I am looking forward to seeing you in September. ☺

Sincerely,

Mrs. Barbara Quick

PS: A good website to check out if you need some help would be www.**calculus-help**.com

AP CALCULUS BC SUMMER ASSIGNMENT

Please check the prerequisites for Calculus BC and study the concepts.

PREREQUISITES FOR CALCULUS BC

Before studying calculus, all students should complete a full four-year preparation of secondary mathematics designed for college-bound students: courses in which they study Algebra, Geometry, Trigonometry, Pre-Calculus (analytic geometry and elementary functions)

Students must be familiar with functions that include:

- Linear
- Polynomial
- Rational
- Exponential
- Logarithmic
- Trigonometric
- Inverse Trigonometric
- Piecewise defined

Students must be familiar with

- the properties of functions,
- the algebra of functions, and
- the graphs of functions.

Students must also understand the language of functions:

- domain and range,
- odd and even,
- periodic,
- symmetry,
- zeroes,
- intercepts, and
- know the exact values of the trigonometric functions of common angles such as 0, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$

Students must be familiar with pre-calculus topics.

1. Functions, Graphs, and Limits

   Analysis of graphs—using technology and calculus to predict and to explain the observed local and global behavior of a function.

2. Limits of functions (including one-sided limits)

   Calculating limits from graphs or tables of data or using algebra

3. Asymptotic and unbounded behavior

   Understanding asymptotes in terms of graphical behavior, describing asymptotic behavior in terms of limits involving infinity, comparing relative magnitudes of functions and their rates of change, and understanding the very significant behavior of functions that the asymptotes reveal.

4. Continuity as a property of functions

   Understanding continuity in terms of limits and geometric understanding of graphs of continuous functions.

5. Parametric, polar, and vector functions.

6. Derivatives

   The concept of the derivative presented geometrically, numerically, and analytically, and is interpreted as an instantaneous rate of change. Understand the definition of a derivative as the limit of the difference quotient. Understand the relationship between differentiability and continuity.

7. Derivative at a point.

   Understand the slope of a curve at a point, the tangent line to a curve at a point, and local linear approximation. Understand instantaneous rate of change as the limit of average rate of change. Approximate rate of change from graphs and tables of values.

8. Derivative as a function

   Corresponding characteristics of graphs of and Relationship between the increasing and decreasing behavior of f(x) and the sign of f’(x).

Partial Fraction Decomposition

Usually with fractions, we add them by creating a common denominator.
Example:

$$\frac{2}{x-3}-\frac{1}{2x+1}\text{ common denominator is }(x-3)(2x+1)$$

$$\frac{2(2x+1)}{(x-3)(2x+1)}-\frac{1(x-3)}{(x-3)(2x+1)}=\frac{4x+2-x+3}{(x-3)(2x+1)}=$$

$$\frac{3x+5}{(x-3)(2x+1)}$$

Now, we will do this in reverse, the process is called partial fraction decomposition.
Express the following as a sum of partial fractions

$$\frac{3x}{(x-1)(x+2)}=\frac{A}{x-1}+\frac{B}{x+2}$$

$$\frac{3x}{(x-1)(x+2)}=\frac{A(x+2)}{(x-1)(x+2)}+\frac{B(x-1)}{(x+2)(x-1)}$$

$$3x=A(x+2)+B(x-1)$$

Let x = 1

$$3(1)=A(1+2)+B(1-1)$$

$$3=3A$$

$$A=1$$

Let x = -2

$$3(-2)=A(-2+2)+B(-2-1)$$

$$-6=-3B$$

$$B=2$$

So, $\frac{3x}{(x-1)(x+2)}=\frac{1}{x-1}+\frac{2}{x+2}$

Express the following as a sum of partial fractions

a. $\frac{2x-1}{(x-3)(x+2)}$

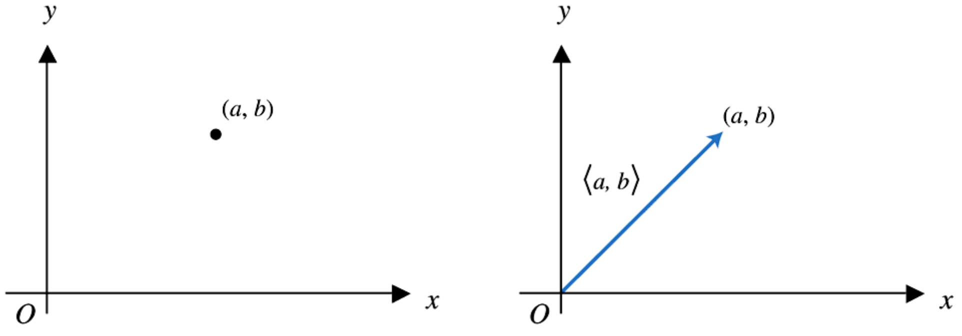
b. $\frac{2x+5}{(x-2)(x+1)}$

c. $\frac{3}{(x-1)(2x-1)}$

d. $\frac{1}{(x-2)(x+4)}$

Vectors

A vector is a directed line segment.

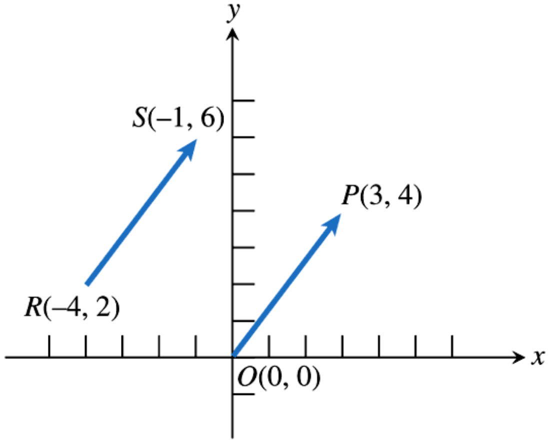


(a) (b)

Two Dimensional Vector

A **two dimensional vector v** is an ordered pair of real numbers, denoted in component form as $\langle a, b \rangle$. The numbers *a* and *b* are the **components** of the vector v. The **standard representation** of the vector $\langle a, b \rangle$ is the arrow from the origin to the point (a, b). The **magnitude** of v is the length of the arrow and the **direction** of v is the direction in which the arrow is pointing. The vector 0 = $\langle 0,0 \rangle$, called the **zero vector**, has zero length and no direction.

Initial Point, Terminal Point, Equivalent



$$\overrightarrow{OP} = \langle 3-0, 4-0 \rangle = \langle 3,4 \rangle$$

$$\overrightarrow{RS} = \langle -1-(-4), 6-2 \rangle = \langle 3,4 \rangle$$

Therefore $\overrightarrow{OP} = \overrightarrow{RS}$

It is important to remember that any two arrows with the same length and pointing in the same direction represent the same vector. For example in the above figure the vector $\langle 3,4 \rangle$ is shown represented by $\overrightarrow{RS}$ with initial point R and terminal point S as well as in standard position $\overrightarrow{OP}$.

Head-Minus Tail Rule

If an arrow has initial point (x_1, y_1) and terminal point (x_2, y_2), it represents the vector $\langle x_2 - x_1, y_2 - y_1 \rangle$

Magnitude

If $\boldsymbol{v}$ is represented by the arrow from (x_1, y_1) to (x_2, y_2), then

$$|\boldsymbol{v}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

If $\boldsymbol{v} = \langle a, b \rangle$, then $|v| = \sqrt{a^2 + b^2}$.

Find the magnitude of v represented by $\overrightarrow{PQ}$, where P=(3, -4) and Q=(5, 2)

$$\overrightarrow{PQ} = \langle 5-3, 2-(-4) \rangle = \langle 2,6 \rangle$$

$$\left\|\overrightarrow{PQ}\right\| = \sqrt{2^2 + 6^2} = \sqrt{40}$$

Standard Unit Vectors

The two vectors i = $\langle 1,0 \rangle$ and j = $\langle 0,1 \rangle$ are the standard unit vectors. Any vector v can be written as an expression in terms of the standard unit vector:

$$\begin{aligned} v &= \langle a, b \rangle \\ &= \langle a,0 \rangle + \langle 0, b \rangle \\ &= a\langle 1,0 \rangle + b\langle 0,1 \rangle \\ &= ai + bj \end{aligned}$$

The vector v = $\langle a, b \rangle$ is expressed as a linear combination a***i*** + b***j*** of the vectors ***i*** and ***j***. The scalars *a* and *b* are the horizontal and vertical components of the vector v.

4. Practice

Write each vector in component form.

a. $\overrightarrow{AB}$ where $A = (-3,2)$ and $B = (-6,7)$

b. $\overrightarrow{PQ}$ where $P = (-4,3)$ and $Q = (-8,-2)$

c. $|p| = 31,\ 54^o$

d. $|r| = 8,\ 45^o$

Find the magnitude for each vector.

e. $\overrightarrow{CD}$ where $C = (2,10)$ and $D = (4,10)$

f. $v = -41i + 3j$

g. $b = \langle \sqrt{43}, -1 \rangle$

h. $p = \langle -2,5 \rangle$

Find the component form of each vector

i. Find -7**f** + 6*v*

$\mathbf{f} = \langle 3,2 \rangle$

$v = \langle 4,-1 \rangle$

j. Find 3f – 7*v*

$\mathbf{f} = \langle 10,7 \rangle$

$v = \langle -6,-11 \rangle$

k. Find 5f + 8g

$\mathbf{f} = \langle -11,8 \rangle$

$g = \langle -3,3 \rangle$

l. Find - u - *v*

$u = \langle -2,1 \rangle$

$v = \langle 12,-9 \rangle$

5. Parametric equations are given. Complete the table and sketch the curve represented by the parametric equations (label the initial and terminal points as well as indicate the direction of the curve).

a. $x = 4\sin t, \quad y = 2\cos t, \quad 0 \le t \le 2\pi$

t	***0***	$\pi/4$	$\pi/2$	$3\pi/4$	π	$3\pi/2$	2π
x							
y							

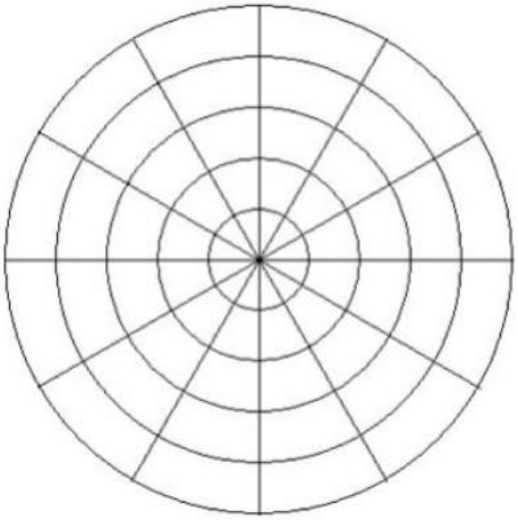
b. $x = 2t - 5, \quad y = 4t - 7, \quad -2 \le t \le 3$

t	***– 2***	***– 1***	***0***	***1***	***2***	***3***
x						
y						

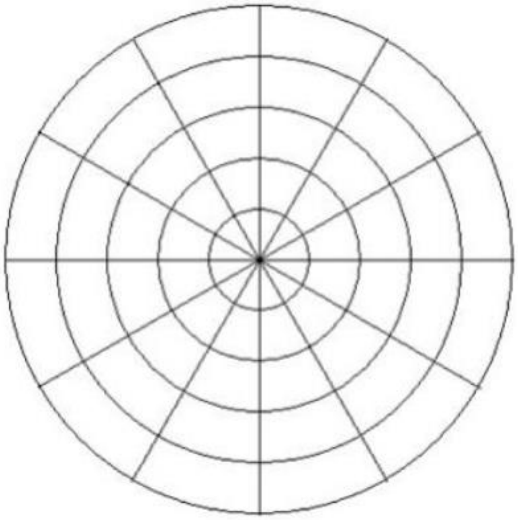
Polar Graphs

Using Desmos, graph and sketch each relation then answer the questions.

$r = 3\cos\theta$



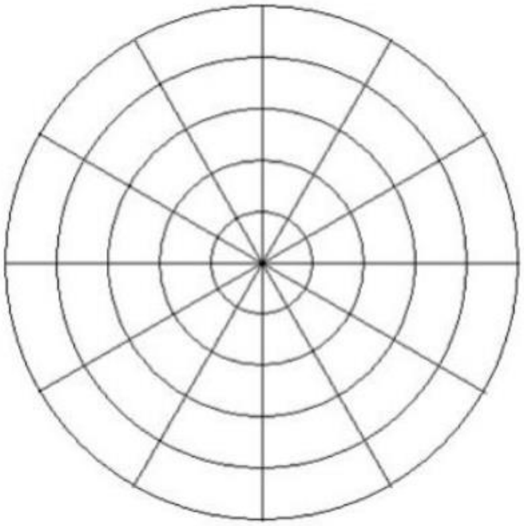
$r = -5\sin\theta$



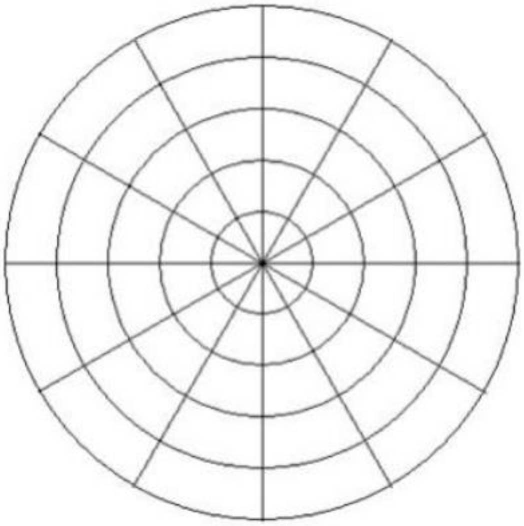
What do these graphs have in common? (things like general shape, axis of symmetry, distances along the axes)

What was different about the graphs? How do these differences link to the equations?

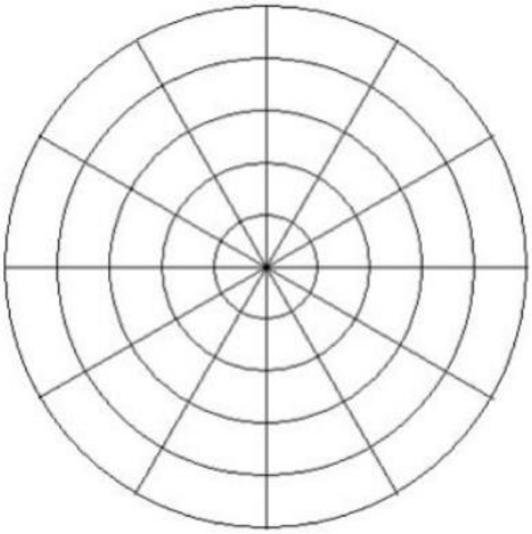
$r = 5\cos(5\theta)$



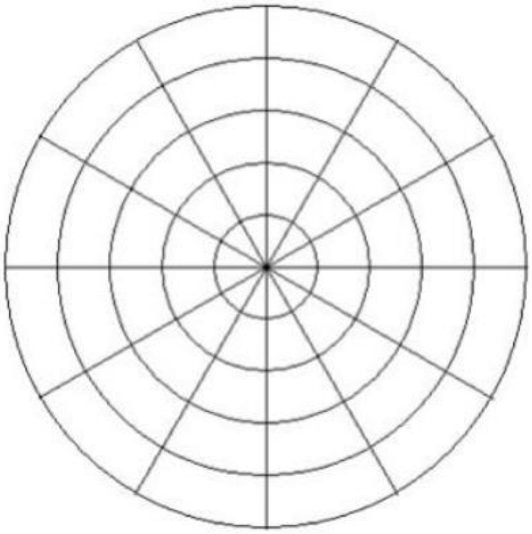
$r = 3\sin(3\theta)$



$r = 3\sin(2\theta)$

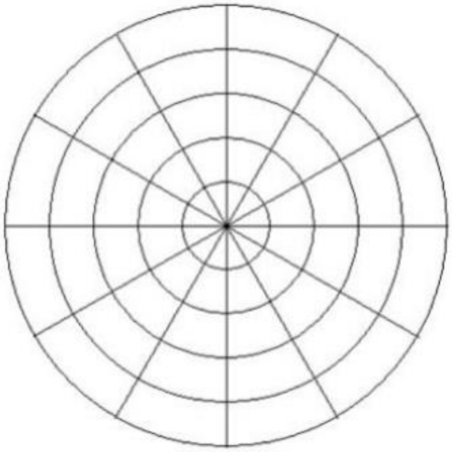


$r = 3\sin(4\theta)$

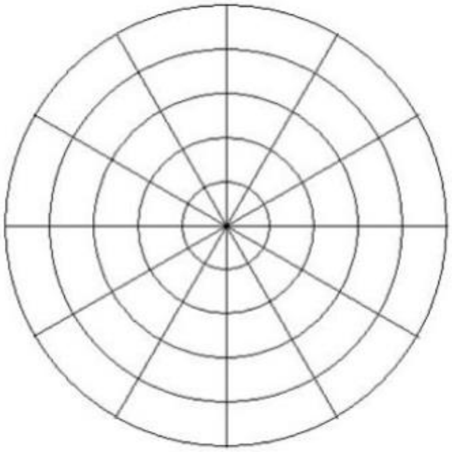


NOTE: We are **MOST** concerned about where the graph intersects the x and y axes, so be sure to draw those points!

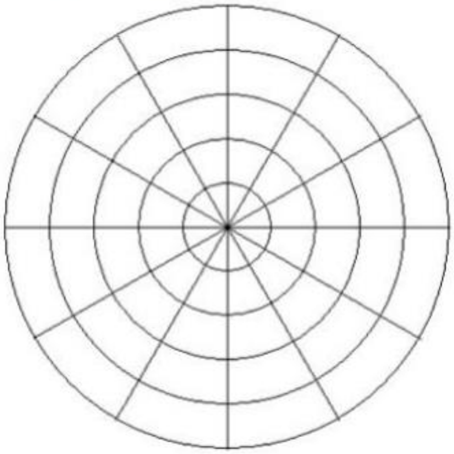
$r = 2 + 3\cos\theta$



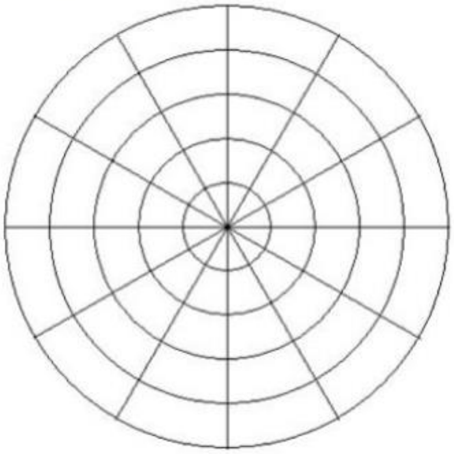
$r = 3 + 3\cos\theta$



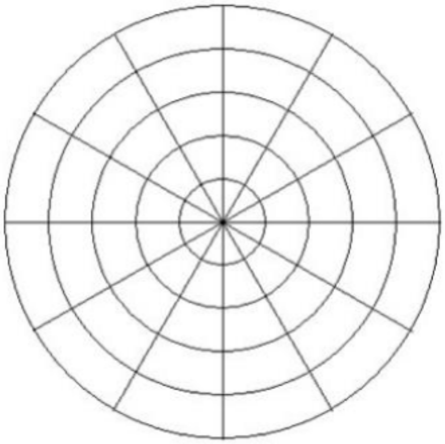
$r = 4 + 3\cos\theta$



$r = 2 - 3\cos\theta$



$r = 3 - 3\cos\theta$



What do these graphs have in common? (things like general shape, axis of symmetry, distances)

What was different about the graphs? How do these differences link to the eqaution?

Review Materials

Limits

1. Find $\lim\limits_{x\to 2}\dfrac{x^3-8}{x-2}$.

(A) 12 (B) 8 (C) 2 (D) does not exist

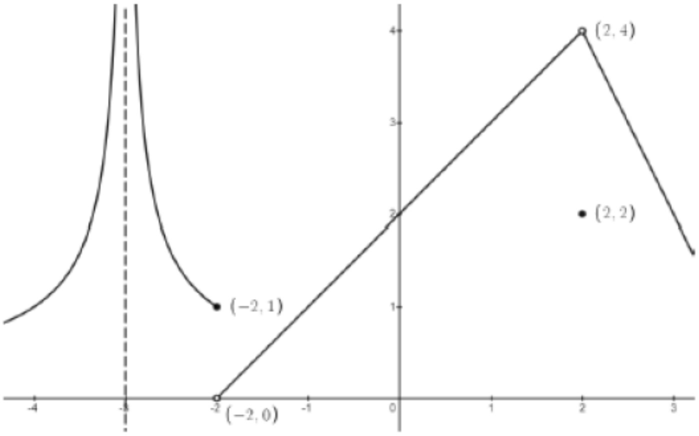
2. The graph of f is shown. At what value of x does f have a removable discontinuity?

(A) -3

(B) -2

(C) 2

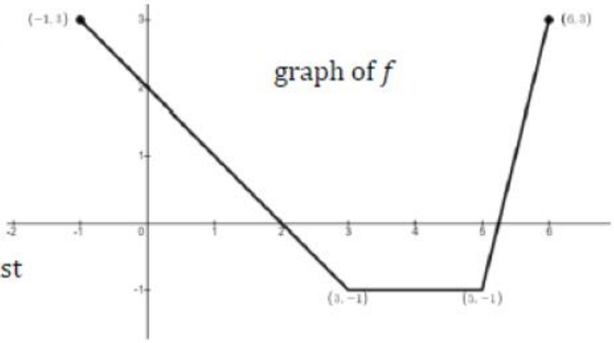
(D) f does not have a removable discontinuity



3. The graph of f consisting of three line segments is shown.

Find $\lim\limits_{x\to 3}\dfrac{\int_{-1}^{x} f(t)dt-4}{2e^{x-3}-2}$.

(A) 0 (B) $-\frac{1}{2}$ (C) $-\frac{5}{2}$ (D) does not exist



4. Given $p(x)=\lim\limits_{h\to 0}\dfrac{\cos(x+h)-\cos x}{h}$. Which of the following is $p'(x)$?

(A) $-\sin x$ (B) $\sin x$ (C) $-\cos x$ (D) $\cos x$

5. Evaluate $\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{2k}{n}\right)^2\frac{2}{n}$.

(A) 9 (B) $8\frac{2}{3}$ (C) 8 (D) $4\frac{2}{3}$

6.
$$f(x)=\begin{cases}\sin(2x) & \text{for } x\le\pi\\ 2x-2\pi & \text{for } x>\pi\end{cases}$$

Let f be the function defined above. Which of the following statements about f is true?

(A) f is continuous and differentiable at $x=\pi$.

(B) f is continuous but not differentiable at $x=\pi$.

(C) f is differentiable but not continuous at $x=\pi$.

(D) f is neither continuous nor differentiable at $x=\pi$.

7.
$$f(x)=\begin{cases}\frac{1}{x^2+2} & \text{for } x<b\\ x^3+5x & \text{for } x\ge b\end{cases}$$

Let f be the function defined above, b is a positive constant. For what value of b, if any is f continuous?

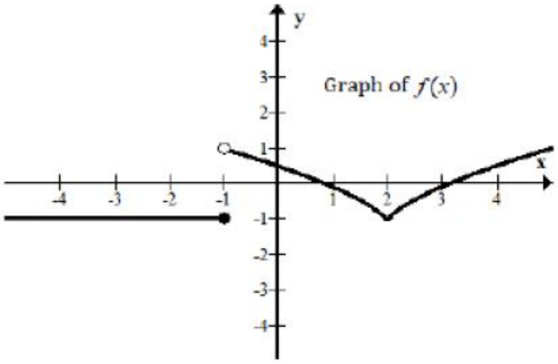
(A) 0.716 (B) 0.498 (C) 0.099 (D) there is no such value

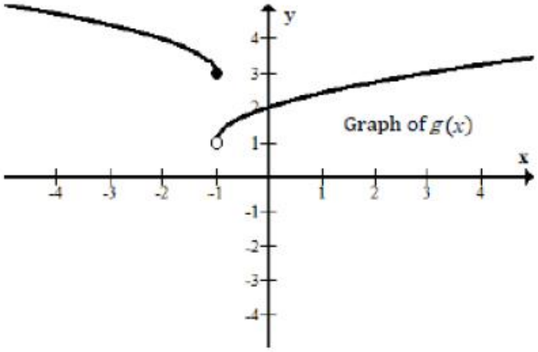
x	1.9	1.99	1.999	2.001	2.01	2.1
$h(x)$	0.556	0.522	0.501	0.498	0.463	0.420

8. The function h is continuous and decreasing for $x>0$. Which of the following values best approximates $\lim_{x\to 2} 5\cdot 9^{3h(x)}$?

(A) 15 (B) 45 (C) 135 (D) 3645

9.





The graphs of the functions f and g are shown above. Find $\lim_{x\to-1}[f(x)+g(x)]$.

(A) 3 (B) 2 (C) 1 (D) -1

10. Consider the function $f(x) = \frac{x^2-3x+2}{x^4-4x^2}$.

(a) Find $\lim_{x\to-2} f(x)$.

(b) Find $\lim_{x\to 0} f(x)$.

(c) Find $\lim_{x\to 2} f(x)$.

(d) Find $\lim_{x\to-\infty} f(x)$.

Derivatives & Chain Rule

1. If $y = \sqrt{x}e^x$, then $\frac{dy}{dx} =$

(A) $\frac{1}{2\sqrt{x}}e^x$ (B) $\frac{1}{2\sqrt{x}}e^x + x\sqrt{x}e^{x-1}$ (C) $\frac{1}{2\sqrt{x}}e^x - \sqrt{x}e^x$ (D) $\frac{1}{2\sqrt{x}}e^x + \sqrt{x}e^x$

2. $\frac{d}{dx}\left(\frac{x-\ln x}{x^2+1}\right)\Big|_{x=1} =$

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) $-\frac{3}{4}$

3. If $x^3 - 2xy - y^2 = -4$, what is $\frac{dy}{dx}$ at the point $(-1, 3)$?

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) $-\frac{3}{4}$

4. $\frac{d}{dx}\left(5\left(\cos\sqrt{x}\right)^2\right) =$

(A) $\frac{-5\cos\sqrt{x}\sin\sqrt{x}}{\sqrt{x}}$ (B) $-10\cos\sqrt{x}\sin\sqrt{x}$ (C) $-10\sin\left(\frac{1}{2\sqrt{x}}\right)$ (D) $-\frac{5\sin\sqrt{x}}{\sqrt{x}}$

5. If $f(x) = e^4 + e^{4x} + x^4 + 4^x$, then $f'(x) =$

(A) $e^4 + 4e^{4x} + 4x^3 + 4^x$ (B) $4e^{4x} + 4x^3 + 4^x \ln 4$ (C) $e^{4x} + 4x^3 + 4^x \ln 4$ (D) $4e^{4x} + 4x^3 + 4^x$

6. If $P(t) = 2\sin t$, then find $P^{(19)}\left(\frac{2\pi}{3}\right)$=

(A) $-\sqrt{3}$ (B) -1 (C) 1 (D) $\sqrt{3}$

7. $\frac{d^2y}{dx^2} = 3x^2 + 7x^3 - 2x^4$; find $\left.\frac{d^5y}{dx^5}\right|_{x=2}$

(A) -54 (B) -48 (C) -42 (D) 0

8. If $y = \arctan(2x)$, $\frac{dy}{dx} =$

(A) $2\operatorname{arcsec}^2(2x)$ (B) $\frac{2}{\sqrt{1-4x^2}}$ (C) $\frac{1}{1+4x^2}$ (D) $\frac{2}{1+4x^2}$

9.

x	$g(x)$	$g'(x)$
2	3	4
3	5	1
4	6	3

The table gives selected values for a differentiable and increasing function g and its derivative. If g^{-1} is the inverse function of g, what is the value of $(g^{-1})'(3)$?

(A) 1 (B) $\frac{1}{4}$ (C) $-1/4$ (D) -1

10. Find the slope of the tangent line of $h(x) = \frac{4}{x} - \sqrt[3]{2x}$ at $x = 4$.

(A) $-\frac{23}{6}$ (B) $-\frac{5}{12}$ (C) $-\frac{1}{3}$ (D) $4\ln 4 - \frac{1}{6}$

Limit Answers

1. **A**
2. **C**
3. **B**
4. **C**
5. **B**
6. **A**
7. **C**
8. **C**
9. **B**
10.
   a. DNE
   b. $-\infty$
   c. $\frac{1}{16}$
   d. 0

Derivative Answers

1. **D**
2. **C**
3. **D**
4. **A**
5. **B**
6. **C**
7. **A**
8. **D**
9. **B**
10. **B**